JEE MAIN 2025

Sample Paper - 4

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

1. There are three subjects in the question paper consisting of Physics (Q. no. 1 to 25), Chemistry (Q, no. 26 to 50), and Mathematics (Q. no. 51 to 75).

2. Each subject is divided into two sections. Section A consists of 20 multiple-choice questions & Section B consists of 5 numerical value-type questions.

3. There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero marks will be awarded for not attempted questions.

4. For Section B questions, 4 marks will be awarded for correct answers and zero for unattempted and incorrect answers.

5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.

6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

PHYSICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

1. A body travels uniformly a distance of $(13.8\pm0.2)\,\text{m}$ in a time (4.0 ± 0.3) s. The velocity of the body within error limits is

A) $(3.45\pm0.2)\text{ms}^{-1}$ B) $(3.45\pm0.3)\text{ms}^{-1}$

C) $(3.45\pm0.4)\text{ms}^{-1}$ D) $(3.45\pm0.5)\text{ms}^{-1}$

2. Statement 1 : In a cyclic process initial and final states are not same

Statement 2 : Initial and final temperatures are same, therefore the change in internal energy is zero

A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.

B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not a correct explanation for Statement – 1.

C) Statement -1 is True, Statement – 2 is False.

D) Statement – 1 is False, Statement – 2 is True.

3. A soap bubble of radius R has uniformly distributed charge Q on it's surface. It's energy is the self-energy of charges & surface energy due to surface tension. In equilibrium, this energy is minimum. Surface tension is S. At equilibrium radius of bubble is R & pressure inside is P. Pressure outside is P_0. Then

A) $R=\left(\dfrac{Q^2}{128\pi^2\epsilon_0 S}\right)^{1/3}$ B) $R=\left(\dfrac{Q^2}{64\pi^2\epsilon_0 S}\right)^{1/3}$ C) $P=P_0-\dfrac{2S}{R}$ D) $P=P_0+\dfrac{2S}{R}$

4. Statement 1 : If a boy standing on a frictionless horizontal surface throws a ball, then he will move backward.

Statement 2 : In the absence of external force the linear momentum of the system remains same

A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.

B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not a correct explanation for Statement – 1.

C) Statement -1 is True, Statement – 2 is False.

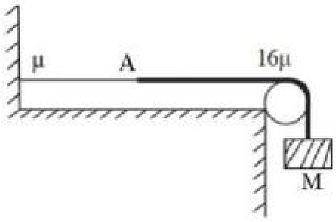
D) Statement – 1 is False, Statement – 2 is True.

5. The work of 146 kJ is performed in order to compress one-kilo mole of gas adiabatically and in this process the temperature of the gas increases by $7°C$. The gas is $(R = 8.3 J mol^{-1} K^{-1})$

A) monoatomic B) diatomic

C) triatomic D) a mixture of monoatomic and diatomic

6. A light string is tied at one end to a fixed support and to a heavy string of equal length L at the other end A as shown in the figure (Total length of both strings combined is $2L$). A block of mass M is tied to the free end of heavy string. Mass per unit length of the strings are μ and 16μ and tension is T. Find lowest positive value of frequency such that junction point A is a node.



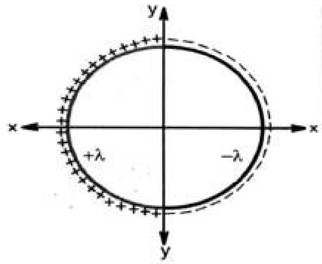
A) $\frac{1}{L}\sqrt{\frac{T}{\mu}}$ B) $\frac{5}{2L}\sqrt{\frac{T}{\mu}}$ C) $\frac{3}{2L}\sqrt{\frac{T}{\mu}}$ D) $\frac{1}{2L}\sqrt{\frac{T}{\mu}}$

7. In the determination of refractive index of material of a parallel sided slab using a travelling microscope the following observations are made. Given least count of microscope is 0.001cm. Then the value of refractive index of material of slab is

Reading of the microscope when focused on									
Sl. no	Mark Made on paper			Mark on paper Through the slab			Particles on top of the glass surface		
	M.S.R. M (cm)	V.S.R. N (cm)	a_1=M+N×L.C. (cm)	M.S.R. M (cm)	V.S.R. N (cm)	a_2=M+N×L.C. (cm)	M.S.R. M (cm)	V.S.R. N (cm)	a_3=M+N×L.C. (cm)
	3.10	19		3.20	15		3.30	10	

A) 2.01 B) 1.50 C) 1.25 D) 2.25

8. A thin ring of radius R metres is placed in x-y plane such that its centre lies at the origin. The half ring in region x<0 carries uniform linear charge density $+\lambda\ C/m$ and the remaining half ring in region x>0 carries uniform linear charge density $-\lambda\ C/m$.



The electric potential (in volts) at point P whose coordinates are $\left(0m, +\frac{R}{2}m\right)$ is

A) Zero B) $\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{2}$ C) $\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{4}$ D) $\frac{1}{4\pi\varepsilon_0}\lambda$

9. A parallel plate capacitor (plate Area: A) connected to battery of emf 'V' and negligible internal resistance, so that one of the plate is made to oscillate and distance between plate varies as $d = d_0 + a\cos(\omega t)$, $a << d_0$. If maximum current observed in circuit is I_0, then the corresponding amplitude of vibration (a) is

A) $\frac{a^2 I_0}{VA\omega \epsilon_0}$ B) $\frac{I_0 d_0}{V\sqrt{A}\omega \epsilon_0}$ C) $\frac{I_0 d_0^2}{VA\omega \epsilon_0}$ D) $\frac{I_0 d_0}{VA\omega \epsilon_0}$

10. Wavelength of first line in Lyman series is λ. The wavelength of first line in Balmer series is

A) $\frac{5}{27}\lambda$ B) $\frac{36}{5}\lambda$ C) $\frac{27}{5}\lambda$ D) $\frac{5}{36}\lambda$

11. Pick out the correct statements of the following

A: If a rigid body is in translational equilibrium, it should be in rotational equilibrium also

B: If a rigid body is in rotational equilibrium, it should be in translational equilibrium Also

C: A body in mechanical equilibrium should be in both translational and rotational equilibrium

D: When a force acting on a body produces turning effect, the force should be a skew vector with respect to the axis of rotation

A) A, B and C are only correct B) B and Care only correct

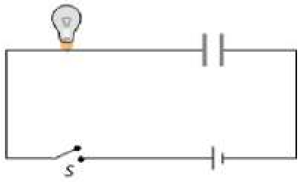
C) C and D are only correct D) Only C is correct

12. Match the entries in the Column I with those in Column II

Column I		Column II	
(A)	Binding energy per nucleon of a nucleus	(P)	Photoelectric effect
(B)	Particle nature of light	(Q)	Nuclear fission and fusion
(C)	Binding energy of products is greater than that of reactants	(R)	Uncontrolled chain reaction
(D)	Atom bomb	(S)	Measure of stability

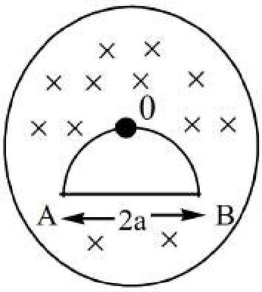
A) A – S; B – P; C – R; D – Q
B) A – S; B – P; C – Q; D - R
C) A – P; B – S; C – Q; D – R
D) A – S; B – Q; C – P; D – R

13. A light bulb, a capacitor and a battery are connected together as shown here, with switch S initially open. When the switch S is closed, which one of the following is true



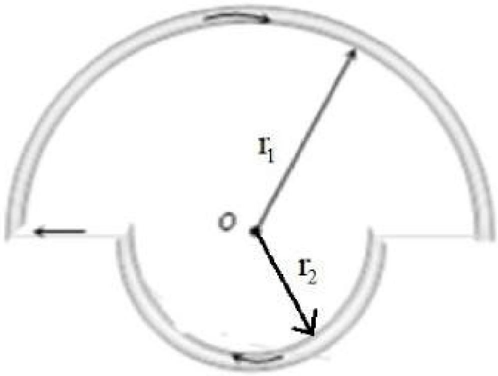
A) The bulb will light up for a short interval of time and then puts off
B) The bulb will light up when the capacitor is fully charged
C) The bulb will not light up at all
D) The bulb will light up and go off at regular intervals

14. In a cylindrical magnetic field B is changing as $B = B_0 + \alpha t$. The value of emf induced in the loop as shown in the figure is______(AOBA is semi circular and α, B_0 are constant values)



A) $\frac{\pi a^2 \alpha}{2}$
B) $a^2 \alpha \left(\frac{\pi}{2} - 1\right)$
C) $a^2 \alpha$
D) $\pi a^2 \alpha$

15. In the figure shown there are two semicircles of radii r_1 and r_2 in which a current i is flowing. The magnetic induction at the centre O will be



A) $\frac{\mu_0 i}{r}(r_1 + r_2)$ B) $\frac{\mu_0 i}{4}(r_1 - r_2)$ C) $\frac{\mu_0 i}{4}\left(\frac{r_1 + r_2}{r_1 r_2}\right)$ D) $\frac{\mu_0 i}{4}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$

16. If a diamagnetic substance is brought near north or south pole of a bar magnet, it is

A) Attracted by the poles

B) Repelled by the poles

C) Repelled by the north pole and attracted by the south pole

D) Attracted by the north pole and repelled by the south pole

17. The potential energy of a particle of mass m is given by

$$U(x) = \begin{cases} E_0; 0 \le x \le 1 \\ 0; \quad x > 1 \end{cases}$$

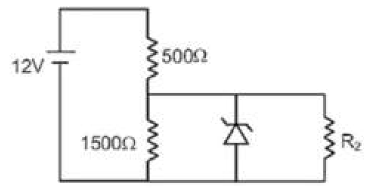
λ_1 and λ_2 are the de-Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$ the ratio $\frac{\lambda_1}{\lambda_2}$ will be

A) 2 B) 1 C) $\sqrt{2}$ D) $\frac{1}{\sqrt{2}}$

18. An AC source of angular frequency ω is applied across a resistor R and a capacitor C in series. The current registered is I. If the frequency of source is changed to $\omega / 3$. (maintaining the same voltage), the current in the circuit is found to be halved. The ratio of reactance to resistance at the original frequency ω is

A) $\sqrt{\frac{3}{5}}$ B) $\sqrt{\frac{2}{5}}$ C) $\sqrt{\frac{1}{5}}$ D) $\sqrt{\frac{4}{5}}$

19. A Zener diode of Zener break – down voltage 10 v is connected as shown in the figure. Current through Zener diode is



A) 4mA B) 0.6mA C) 6mA D) Zero

20. With a concave mirror, an object is placed at a distance x_1 from the principal focus, on the principal axis. The image is formed at a distance x_2 from the principal focus. The focal length of the mirror is

A) $x_1 x_2$ B) $\frac{x_1 + x_2}{2}$ C) $\sqrt{\frac{x_1}{x_2}}$ D) $\sqrt{x_1 x_2}$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

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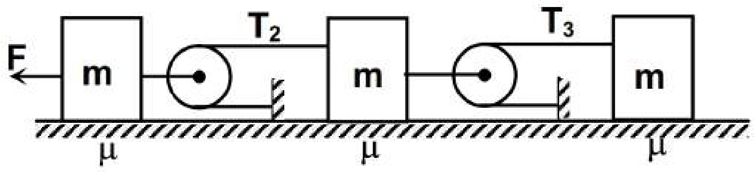
21. A particle is moving in a circular path with velocity (in ms^{-1}) varying with time as $v = 1.5t^2 + 2t$. The radius of the circular path is 2cm. Then the angular acceleration of the particle at $t = 2\sec$ is ______(in rad/sec^2)

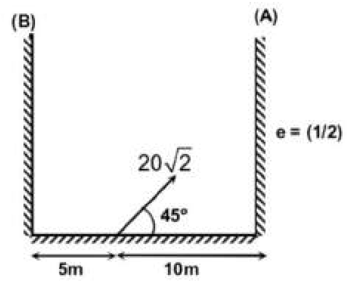
22. On a rough table, three blocks (including the first block) are placed as shown in the figure. Mass of each block is m and coefficient of friction for each block is μ. A force F is applied on the first block so as to move the system. If the minimum value of F required is $n\mu mg$, find n.



23. In a meter bridge a 30Ω resistance is connected in the left gap and a pair of resistances P and Q in the right gap. Measured from the left, the balance point is 37.5 cm, when P and Q are in series and 71.4 cm when they are parallel. Find the value of $\left(\frac{P}{Q}\right)^2$?

24. A ball is projected with speed $20\sqrt{2}$ m/s at an angle of $45°$ with horizontal. It collides first with the right wall A (e = 1/2) and then with the left wall B, and finally returns to the projection point. Then find the coefficient of restitution between ball and wall B.

$(g = 10 \text{ m/s}^2)$



25. The ratio of amplitudes of two coherent waves in Young's double – slit experiment is $\frac{A_1}{A_2} = \frac{1}{3}$. What is the ratio of maximum and minimum intensities of fringes?

CHEMISTRY **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

26. One litre of mixture of oxygen and ozone at STP (1atm, 273K) was allowed to react with excess of acidified solution of KI. Released iodine required 80 mL of M/20 sodium thiosulphate for titration. Volume of oxygen in original mixture at STP will be

$$2KI + H_2O + O_3 \rightarrow 2KOH + I_2 + O_2$$

$$I_2 + 2Na_2S_2O_3 \rightarrow Na_2S_4O_6 + 2NaI$$

A) 955.2 mL B) 800 mL C) 0.056 L D) 955.2 L

27. Consider the Assertion and Reason given below

Assertion: If diethyl ether and water are used for differential extraction, diethyl ether would stay as an upper layer and water would stays as lower layer in separating funnel.

Reason: The density of diethyl ether is less than water

Choose the correct answer from the following

A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

B) Both (A) and (R) are correct and (R) is the correct explanation of (A)

C) (A) is correct but (R) is wrong

D) (A) and (R) both are wrong

28. $(X) + K_2CO_3 + Air \xrightarrow{heat} (Y) + CO_2 \uparrow$

$(Y) + Cl_2 \longrightarrow (Z)\, Purple$

Which of the following is correct?

A) X= black, MnO_2, Y=Blue, K_2CrO_4,Z= $KMnO_4$

B) X= green, Cr_2O_3, Y= yellow, K_2CrO_4, $Z = K_2Cr_2O_7$

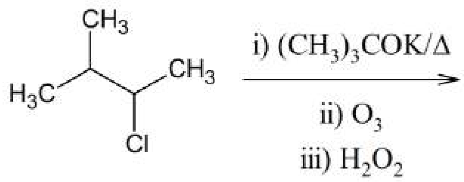
C) X= black, MnO_2, Y= green, K_2MnO_4, Z= $KMnO_4$

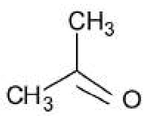
D) X= black, Bi_2O_3, Y= colourless $KBiO_2$, $Z = KBiO_3$

29. Maximum number of different spectral lines observed when a sample contains 1000 hydrogen atoms in third excited state undergoes transition of electrons to ground state are

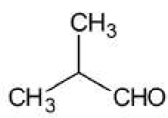
A) 3 B) 4 C) 5 D) 6

30. The major end products of the following reaction are

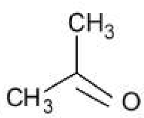


A) 

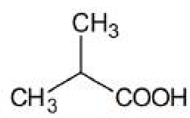
+ CH_3CHO

B) 

+ HCHO

C) 

+ CH_3COOH

D) 

+ HCOOH

31. Consider that a d^6 metal ion (M^{2+}) forms a complex with aqua ligands, and the spin only magnetic momentum of the complex is 4.90 BM. The geometry and the crystal field stabilization energy of the complex is

A) Tetrahedral and $-1.6\Delta_t + 1$ P B) Octahedral and $-2.4\Delta_0 + 1$ P

C) Octahedral and $-1.6\Delta_0$ D) Tetrahedral and $-0.6\Delta_t$

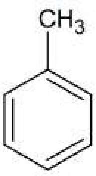
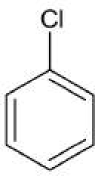
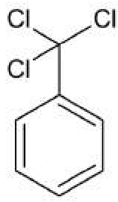
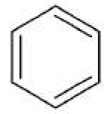
32. Consider the following equilibria:

$$A_{(s)} \rightleftharpoons B_{(g)} + C_{(g)} \qquad (K_p)_1 = x$$
$$B_{(g)} \rightleftharpoons D_{(g)} \qquad (K_p)_2 = 2$$

Initially only $A_{(s)}$ was present and final total pressure at equilibrium is 20 atm. Equilibrium constant (x) is found to be

A) 100 B) 33.33 C) 50 D) 24

33. The correct order of reactivity towards nitration using HNO_3/H_2SO_4 mixture for the given compounds is

I) 
II) 
III) 
IV) 

A) I>II>IV>III B)I>IV>II>III C)II>I>IV>III D)II>IV>I>III

34. Which of the following order is Correctly matched

A) Melting point $NH_3 > PH_3 > AsH_3 > SbH_3$

B) HEH bond angle $NH_3 > PH_3 > AsH_3 > SbH_3$ (E= central atom)

C) Boiling point $NH_3 < PH_3 < AsH_3 < SbH_3$

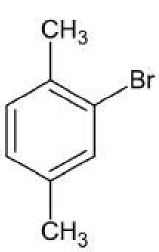
D) E –H Distance /pm $NH_3 > PH_3 > AsH_3 > SbH_3$ (E= central atom)

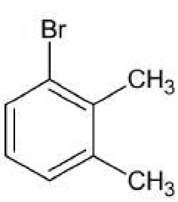
35. Which of the following metals can't be detected in the laboratory through flame test

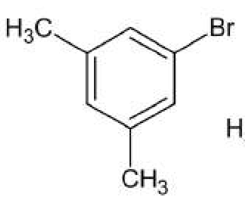
A) Na B) Mg C) K D) Ba

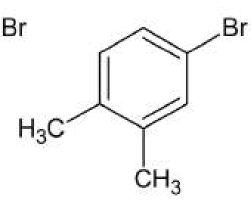
36. Which two isomeric compounds give same tri carboxylic acid after completion of the below scheme of reactions?

$$\text{Isomeric compound} \xrightarrow[\text{iii) } H^+ \text{ iv) } KMnO_4]{\text{i) Mg / Dry ether ii) Anhy. } CO_2} \text{Tri carboxylic acid}$$


I


II


III


IV

A) I and II B) III and IV C) I and IV D) II and III

37. Consider the Assertion and Reason given below

Assertion: C-O bond order in the complexes is

$\left[Cr(CO)_5 PF_3\right] > \left[Cr(CO)_5 PCl_3\right] > \left[Cr(CO)_5 PMe_3\right]$

Reason: Order of Electrons in the ABMO of CO

is $\left[Cr(CO)_5 PF_3\right] < \left[Cr(CO)_5 PCl_3\right] < \left[Cr(CO)_5 PMe_3\right]$

Choose the correct answer from the following

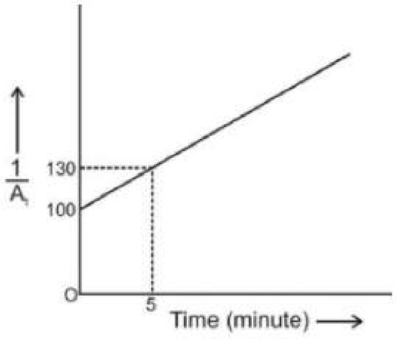
A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

B) Both (A) and (R) are correct and (R) is the correct explanation of (A)

C) (A) is correct but (R) is wrong

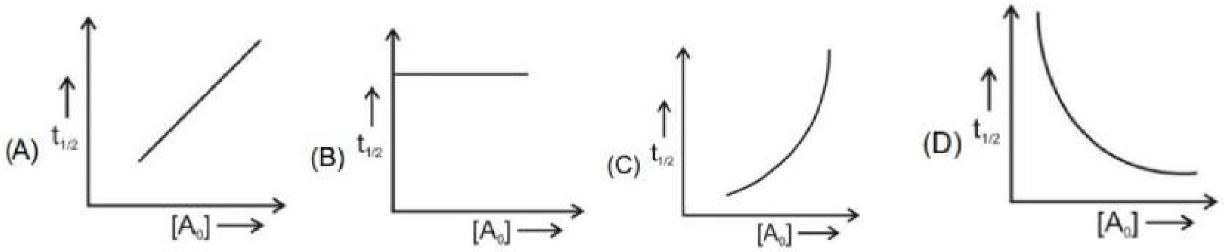
D) (A) and (R) both are wrong

38. Consider a chemical reaction $nA \rightarrow products$ for which following graphical representation is given:

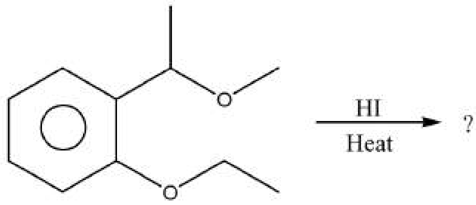


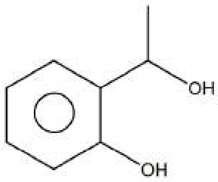
A_t is concentration of A at time t in mol L^{-1}.

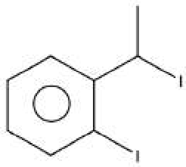
Which of the following the most appropriate graph indicating relation between half life and initial concentration of A.

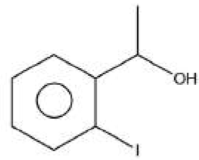


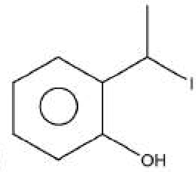
39. The major product formed in the following reaction is



A) 

B) 

C) 

D) 

40. Match the columns I,II and III and mark the appropriate choice

	Column-I		Column-II		Column-III
a)	Bromine	i)	Noble metal	p)	Amalgam
b)	Gold	ii)	Crystalline non-metal	q)	$4s^2 4p^5$
c)	Mercury	iii)	Liquid non-metal	r)	Transition metal
d)	Iodine	iv)	Liquid metal	s)	Violet

A) a – ii, p, b – i, s c – iii, q d – iv, r

B) a – iii, q b – i,r c – iv, p d – ii, s

C) a – i, s b – ii, p c – iv, r d – iii, q

D) a – iv, q b – i,r c – iv, p d – ii, s

41. Which of the following is correct order for the mentioned property

i) SH_4=SiF_4=$SiCl_4$ (Bond angle)

ii)Vinyl chloride >Allyl Chloride (C-Cl Bond length)

iii) $Na_2O_2 > KO_2 > O_3$ (O – O Bond order)

1) i,ii,iii 2) i,ii only 3) i only 4) ii,iii only

42. Benzaldehyde shows positive

(i) Tollen's test (ii) Fehiling's test (iii) lodoform test (iv) DNP test

A) iii B) i and iv C) ii and iv D) ii and iii

43. Choose the correct statement(s), regarding molecular orbitals

i)If inter nuclear axis is z-axis, bonding molecular orbital obtained from $2p_x$ and $2p_x$ orbitals are not symmetrical around the bond axis.

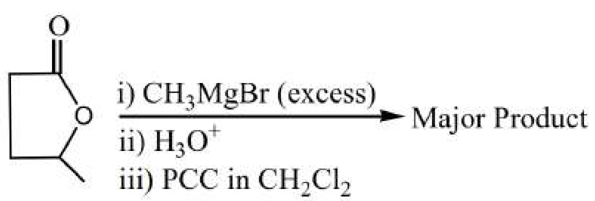
ii)The combining atomic orbitals must have the same symmetry about the molecular axes.

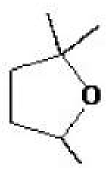
iii)Sigma bonding molecular orbital are symmetrical around bond axis where as pi bonding molecular orbitals are unsymmetrical.

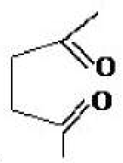
iv)Stability order of orbitals is Molecular orbital > Atomic orbital > Anti bonding Molecular orbital

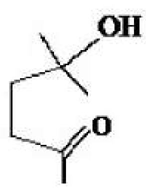
A) i,ii,iii only B) i and iv only C) ii and iv only D) i,ii,iii and iv

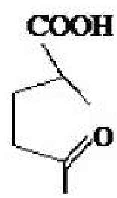
44. The major product of following reaction is



A) 

B) 

C) 

D) 

45. Which amine gives solid with benzenesulphonylchloride and that solid is insoluble in KOH ?

A) $CH_3CH_2NH_2$ B) CH_3NHCH_3 C) $(CH_3)_3N$ D) $CH_3CH(NH_2)CH_3$

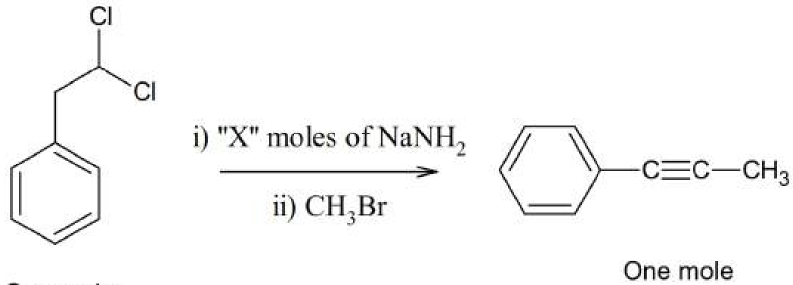
SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**

Marking scheme: +4 for correct answer, -1 in all other cases.

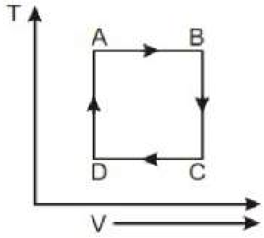
46. For a unielectronic species, the radial component of Schrodinger wave equation for ns orbital is given as $\psi = \frac{2}{81\sqrt{3}}\left(\frac{Z}{a_o}\right)^{\frac{3}{2}}(27-18\sigma+2\sigma^2)e^{-\frac{a}{3}}$ $[\sigma = \frac{Zr}{a_o}]$, value of principle quantum number(n) will be

47.



The value of [X] is

48. Following reversible processes are performed for one mole a monoatomic ideal gas :



Select the number of CORRECT statements among the following:

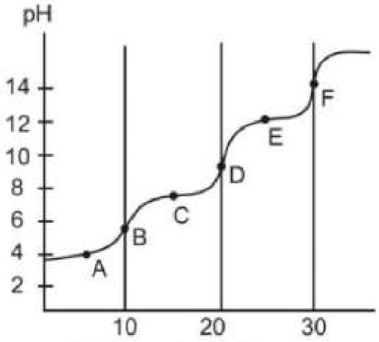
P : $(\Delta E)_{A\to B} = 0, (\Delta H)_{A\to B} = 0$ Q : $(w)_{B\to C} = 0$

R : $(\Delta S)_{C\to D} < 0$ S : $(q)_{A\to B} > 0$

49. 5 mol of KI is mixed with 1 mol of $HgCl_2$ in 3.5 L of water. Magnitude of freezing point of the solution will be nearest to $(KI + HgCl_2 \rightarrow K_2HgI_4 + KCl)$

$(k_f = 1.86\,K.kg.mol^{-1})$

50. Neutralisation curve for orthophosphoric acid and NaOH is observed as



A, C, E → Half equivalence point
B, D, F → neutralisation point

How many of the following statements are CORRECT ?

P : At points A, C and E, $pH = pK_{a_1}$, $pH = pK_{a_2}$, $pH = pK_{a_3}$ respectively.

Q : At point B and D, $pH = \frac{pK_{a_1} + pK_{a_2}}{2}$ and $pH = \frac{pK_{a_2} + pK_{a_3}}{2}$ respectively.

R : At point " C ", $H_2PO_4^-$ and HPO_4^{2-} are in equal amount.

MATHEMATICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

51. Statement – 1: If a function y = f(x) is symmetric about y = x, then f(f(x)) = x

Statement – 2: If $f(x)=\begin{cases} x & : \text{x is rational} \\ 1-x & : \text{x is irrational} \end{cases}$, then f(f(x)) = x

A) Statement-1is True, Statement-2 is True

B) Statement-1 is False, Statement-2 is False

C) Statement-1 is True, Statement-2 is False

D) Statement-1 is False, Statement-2 is True

52. Let $P=\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q=[q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$, If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to

A) 13 B) 15 C) 17 D) 21

53. Statement – I: If $\alpha = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$, $p = \alpha + \alpha^2 + \alpha^4, q = \alpha^3 + \alpha^5 + \alpha^6$, then the equation where roots are p and q is $x^2 + x + 2$.

Statement – II : If α is a root of $Z^7 = 1$, then $1 + \alpha + \alpha^2 + ... + \alpha^6 = 0$

A) Statement 1 is true, Statement – 2 is true

B) Statement 1 is false, Statement 2 is false

C) Statement 1 is true; Statement 2 is false

D) Statement 1 is false; Statement 2 is true

54. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is

A) 1492 B) 1493 C) 1490 D) 1491

55. Match the following:

Column I		Column II	
(A)	Number of triangle that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly one side common with the polygon is	(p)	75
(B)	Number of triangles that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly 2 sides common with the polygon is	(q)	110
(C)	Number of quadrilaterals that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly 2 sides common with the polygon	(r)	60
(D)	Number of quadrilaterals that can be made using the vertices of a polygon of 10 sides as their vertices had having 3 sides common with the polygon is	(s)	10

A) A-r, B-s, C-p. D-q B) A-s. B-r, C-p, D-s

C) A-r, B-s, C-p, D-s D) None of these

56. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0,1)$.

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to

A) 9 B) 3 C) 7 D) 6

57. In Let the circumcentre of a triangle with vertices A(a, 3), B(b, 5) and C(a, b), ab > 0 be P(1, 1). If the line AP intersects the line BC at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to

A) 2 B) $\frac{4}{7}$ C) $\frac{2}{7}$ D) 4

58. The number of real solutions of the equation $e^{4x}+4e^{3x}-58e^{2x}+4e^{x}+1=0$ is

A) 4 B) 6 C) 2 D) 8

59. A wire of length 20 m is to be cut into two pieces. A piece of length ℓ_1 is bent to make a square of area A_1 and the other piece of length ℓ_2 is made into a circle of area A_2. If $2A_1+3A_2$ is minimum then $(\pi\ell_1):\ell_2$ is equal to

A) 6 : 1 B) 3 : 1 C) 1 : 6 D) 4 : 1

60. Let $f(x)=4x^3-11x^2+8x-5, x\in R$. Then f:

A) has a local minima at $x=\frac{1}{2}$. B) has a local minima at $x=\frac{3}{4}$

C) is increasing in $\left(\frac{1}{2},\frac{3}{4}\right)$ D) is decreasing in $\left(\frac{1}{2},\frac{4}{3}\right)$

61. Let for a triangle ABC,

$\overline{AB}=-2\hat{i}+\hat{j}+3\hat{k}$

$\overline{CB}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$

$\overline{CA}=4\hat{i}+3\hat{j}+\delta\hat{k}$

If $\delta>0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overline{CB}.\overline{CA}$ is equal to

A) 60 B) 120 C) 108 D) 54

62. If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

A) 16 B) 6 C) 12 D) 15

63. If $15\sin^4\alpha+10\cos^4\alpha=6$, for $\alpha\in R$, then the value of $27\sec^6\alpha+8\operatorname{cosec}^6\alpha$ is equal to:

A) 350 B) 250 C) 400 D) 500

64. Let $S_1=\left\{x\in R-\{1,2\}:\dfrac{(x+2)(x^2+3x+5)}{-2+3x-x^2}\ge 0\right\}$ and $S_2=\{x\in R:3^{2x}-3^{x+1}-3^{x+2}+27\le 0\}$. Then, $S_1\cup S_2$ is equal to:

A) $(-\infty,-2]\cup(1,2)$ B) $(-\infty,-2]\cup[1,\ 2]$ C) $(-2,1]\cup(2,\infty)$ D) $(-\infty,2]$

65. The integral $\int\left(\dfrac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to (where C is a constant of integration):

A) $\tan x-\dfrac{x\sec x}{x\sin x+\cos x}+C$

B) $\sec x+\dfrac{x\tan x}{x\sin x+\cos x}+C$

C) $\sec x-\dfrac{x\tan x}{x\sin x+\cos x}+C$

D) $\tan x+\dfrac{x\sec x}{x\sin x+\cos x}+C$

66. The area of the region $S=\left\{(x,y):y^2\le 8x, y\ge\sqrt{2}x, x\ge 1\right\}$ is

A) $\dfrac{13\sqrt{2}}{6}$ B) $\dfrac{11\sqrt{2}}{6}$ C) $\dfrac{5\sqrt{2}}{6}$ D) $\dfrac{19\sqrt{2}}{6}$

67. Let $y=y(x), y>0$ be a solution curve of the differential equation $(1+x^2)dy=y(x-y)dx$. If $y(0)=1$ and $y(2\sqrt{2})=\beta$, then

A) $e^{3\beta^{-1}}=e(3+2\sqrt{2})$

B) $e^{\beta^{-1}}=e^{-2}(5+\sqrt{2})$

C) $e^{\beta^{-1}}=e^{-2}(3+2\sqrt{2})$

D) $e^{3\beta^{-1}}=e(5+\sqrt{2})$

68. The number of terms common to the two A.P,'s 3, 7, 11, …., 407 and 2, 9, 16, …., 709 is ___________.

A) 7 B) 14 C) 21 D) 28

69. Consider the following frequency distribution :

Class:	0-6	6-12	12-18	18-24	24-30
Frequency :	a	b	12	9	5

If mean $=\dfrac{309}{22}$ and median = 14, then the value $(a-b)^2$ is equal to _________.

A) 2 B) 6 C) 4 D) 8

70. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to

A) 15 B) 50 C) 45 D) 30

SECTION-II

(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**

Marking scheme: +4 for correct answer, -1 in all other cases.

71. Let z be the complex numbers which satisfy $|z+5| \le 4$ and $z(1+i)+\bar{z}(1-i) \ge -10, i=\sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha+\beta)$ is __________.

72. Consider a matrix $A=\begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$ where α,β,γ are three distinct natural numbers.

If $\dfrac{\det(\text{adj}(\text{adj}(\text{adj}(\text{adj}A))))}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}} = 2^{32}\times 3^{16}$, then the number of such 3-triples (α,β,γ) is __________.

73. The total number of 4-digit numbers whose greatest common division with 54 is 2, is

74. The probability distribution of X is :

X	0	1	2	3
P(X)	$\frac{1-d}{4}$	$\frac{1+2d}{4}$	$\frac{1-4d}{4}$	$\frac{1+3d}{4}$

For the minimum possible value of d, sixty times the mean of X is equal to ______

75. If the system of equations.

$x+y+z=16$

$2x+5y+\alpha z=\beta$

$x+2y+3z=14$

Has infinitely many solutions, then $\alpha+\beta$ is equal to

SOLUTIONS
PHYSICS

1. Here, $S=(13.8\pm0.2)m$

and $t=(4.0\pm0.3)$ sec

Expressing it in percentage error, we have,

$$S=13.8\pm\frac{0.2}{13.8}\times100\%=13.8\pm1.4\%$$

and $$t=4.0\pm\frac{0.3}{4}\times100\%=4\pm7.5\%$$

$$\because V=\frac{s}{t}=\frac{13.8\pm1.4}{4\pm7.5}=(3.45\pm0.3)m/s$$

2. Conceptual

3. $$\frac{Q^2}{2\times4\pi\in_0 R}+8\pi R^2S=U\Rightarrow\frac{Q^2}{8\pi\in_0 R}+8\pi R^2S=U$$

$$\frac{dU}{dR}=0$$

$$\Rightarrow R^3=\frac{Q^2}{8\pi\in_0\times16\pi S}$$

4. Conceptual

5. $$\eta=1-\frac{T_2}{T_1},\ \eta'=1-\frac{T_2-100}{T_1-100}\Rightarrow\eta'>\eta$$

6.

$$\frac{P_0V_0}{T_0}=\frac{P_0(1-\alpha h)}{T_0\sqrt{1-\alpha h}}\Rightarrow V=\frac{V_0}{\sqrt{1-\alpha h}}$$

$$dv=\frac{-1}{2}V_0(-\alpha)\frac{1}{1-\alpha h^{3/2}}dh$$

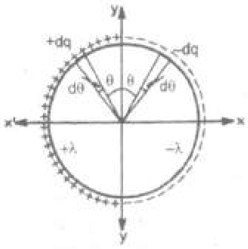
$$\int dw=\int_0^h P\,dv=\int_0^h P_0(1-\alpha h)\frac{v_0\alpha}{2}\frac{1}{(1-\alpha h)^{3/2}}dh$$

$$=\frac{P_0V_0\alpha}{2}\int\frac{1}{(1-\alpha h)^{1/2}}dh=\frac{P_0V_0\alpha}{2}\left[\frac{(1-\alpha h)^{1/2}}{1/2(-\alpha)}\right]_0^h$$

$$=P_0V_0\left(\sqrt{1-\alpha h}\right)$$

7. $$\mu=\frac{3.310-3.119}{3.310-3.215}=\frac{0.191}{0.095}=2.01$$

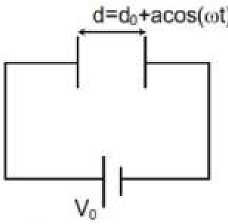
8. Consider two small elements of ring having charges +dq symmetrically located about y-axis



The potential due to this pair at any point on y-axis is zero. The sum of potential due to all such possible pairs is zero at all points on y-axis. Hence potential at $P\left(0,\frac{R}{2}\right)$ is zero

9. $I=\frac{d}{dt}Q=\frac{d}{dt}(CV)=V\frac{d}{dt}\left(\frac{\epsilon_0 A}{d_0+a\cos\omega t}\right)$

$$I=\frac{V\epsilon_0 A[a\omega\sin\omega t]}{(d_0+a\cos\omega t)^2}=\frac{V\epsilon_0 A[a\omega\sin\omega t]}{[d_0+a\cos\omega t]^2}$$



When $\sin\omega t=1$, $\cos\omega t=0$ 'I' becomes maximum

$$\therefore I_0=\frac{V\epsilon_0 A\omega a}{d_0^2}$$

$$a=\frac{I_0 d_0^2}{V\epsilon_0 A\omega}$$

10. $\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)\Rightarrow\frac{1}{\lambda'}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$

$$\frac{\lambda'}{\lambda}=\frac{\left(\frac{1}{1^2}-\frac{1}{2^2}\right)}{\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}=\frac{3/4}{5/36}=\frac{3}{4}\times\frac{36}{5}=\frac{27}{5}$$

11. Theorical concept
12. Conceptual
13. Initially when key is closed, the capacitor acts as short-circuit, so bulb will light up. But finally the capacitor becomes fully charged, so it will act as open circuit, so bulb will not glow
14. $\frac{\pi a^2\alpha}{2}=\int_{BOA}\vec{E}.d\vec{l}+\int_{AB}\vec{E}.d\vec{l}$
15. $\frac{\pi a^2\alpha}{2}=\int_{BOA}\vec{E}.d\vec{l}+a^2\alpha \quad\Rightarrow\quad \int_{BOA}\vec{E}.d\vec{l}=a^2\alpha\left(\frac{\pi}{2}-1\right)$ parts will be in the same direction perpendicular to

$$\therefore B=B_1+B_2=\frac{\mu_0 i}{4r_1}+\frac{\mu_0 i}{4r_2}=\frac{\mu_0 i}{4}\left(\frac{r_1+r_2}{r_1 r_2}\right)\otimes$$

16. Diamagnetic material shows weak repulsion towards any magnetic pole
17. $K.E.=2\,E_0-E_0=E_0(\text{for } 0\le x\le 1)\Rightarrow\lambda_1=\frac{h}{\sqrt{2mE_0}}$

$$K.E.=2E_0(\text{for } x>1)\Rightarrow\lambda_2=\frac{h}{\sqrt{4mE_0}}\Rightarrow\frac{\lambda_1}{\lambda_2}=\sqrt{2}$$

18. At angular frequency ω, the current in RC circuit is given by

$$i_{rms}=\frac{V_{rms}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \qquad \ldots(i)$$

Also $\dfrac{i_{rms}}{2}=\dfrac{V_{rms}}{\sqrt{R^2+\left(\dfrac{1}{\frac{\omega}{3}C}\right)^2}}=\dfrac{V_{rms}}{\sqrt{R^2+\dfrac{9}{\omega^2C^2}}}$ ...(ii)

From equation (i) and (ii), we get

$$3R^2=\frac{5}{\omega^2C^2}\Rightarrow\frac{\frac{1}{\omega C}}{R}=\sqrt{\frac{3}{5}}\Rightarrow\frac{X_C}{R}=\sqrt{\frac{3}{5}}$$

19. Conceptual

20. $\dfrac{1}{u}+\dfrac{1}{v}=\dfrac{1}{f}$

$$\frac{1}{f-x_1}+\frac{1}{f-x_2}=\frac{1}{f}$$

or $\dfrac{f-x_2+f-x_1}{(f-x_1)(f-x_2)}=\dfrac{1}{f}$

or $f^2-fx_2-fx_1+x_1x_2=2f^2-f(x_1+x_2)$

or $f^2=x_1x_2$ or $f=\sqrt{x_1x_2}$

21. $v=1.5t^2+2t$

$$a=\frac{dv}{dt}=3t+2$$

$$\alpha=\frac{a}{r}=\frac{3t+2}{0.02}=\frac{8}{0.02}=400$$

22. $T_3=\mu mg$

$T_2=\mu mg+2T_3=3\mu mg$

$F=\mu mg+2T_2=7\mu mg$

23. **I^st case**

$$\frac{30}{P+Q}=\frac{l}{(100-l)}\Rightarrow\frac{30}{P+Q}=\frac{37.5}{(100-37.5)}$$

$$\frac{30}{P+Q}=\frac{37.5}{62.5}\Rightarrow P+Q=\frac{30\times 62.5}{37.5}$$

$P+Q=50$ ...(i)

II^nd case

$$\frac{30}{\frac{PQ}{P+Q}}=\frac{l}{(100-l)}$$

$$\frac{30(P+Q)}{PQ}=\frac{71.4}{(100-71.4)}$$

$$\frac{30\times 50}{PQ}=\frac{71.4}{28.6}\Rightarrow \quad PQ=\frac{30\times 50\times 28.6}{71.4}$$

$$P\approx 600 \qquad \ldots(\text{ii})$$

So, from Eqs. (i) and (ii)

$P=30\,\Omega$ and $Q=20\,\Omega$

24. Conceptual
25. Conceptual

CHEMISTRY

27. The density of diethyl ether is less than water

28. $$\underset{(X)}{2MnO_2} + 2K_2CO_3 + \underset{(air)}{O_2} \xrightarrow{\Delta} \underset{\substack{green \\ (y)}}{2K_2MnO_4} + 2CO_2(g)$$

$$\underset{(Y)}{2K_2MnO_4} + Cl_2 \longrightarrow \underset{(y)Pink}{2KMnO_4} + 2KCl$$

30. Oxidative ozonolysis

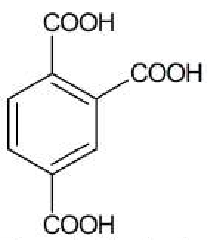
31. $e_g{}^3 t_{2g}{}^3$

33. Reactivity directly proportional to electron density

34. b)HEH bond angle $NH_3(107.8) > PH_3(93.6) > AsH_3(91.8) > SbH_3(91.3)$ (E= central atom)

35. Na gives Golden yellow
K gives Lilac
Ba Green

36.



38. Phenyl alkyl ether become phenol

40.

	Column-I		Column-II		Column-III
a)	Bromine	iii)	Liquid non-metal	q)	$4s^2 4p^5$
b)	Gold	i)	Noble metal	r)	Transition metal
c)	Mercury	iv)	Liquid metal	p)	Amalgam
d)	Iodine	ii)	Crystalline non-metal	s)	Violet

41. All bond angles are same

42. Benzaldehyde cannot give Fehiling's test

43. Refer NCERT

44. PCC cannot oxidise ter. alcohols

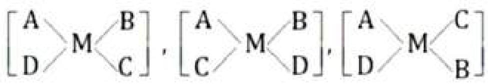
45. Secondary amine

47. Acetyloide anion acts as donor

48. All carbons are Sp^3

49. Two for self and two for crossed

50. Complex is square planar



MATHS

51. (i) y = f(x) is symmetric about y = x ⇒ x = f(y)

∴ f(f(x)) = f(y) = x

∴ statement 1 is true

(ii) $f(x) = \begin{cases} x & , \ x \text{ is rational} \\ 1-x & , \ x \text{ is irrational} \end{cases}$ is

Symmetric about y = x

∴ f(f(x)) = x

52. Given that $PQ = kI$

$|P|.|Q| = k^3$

$\Rightarrow |P| = 2k \neq 0 \Rightarrow P$ is an invertible matrix

$\because PQ = kI \quad \therefore Q = kP^{-1}I \qquad [\because P^{-1}P = I]$

$\therefore Q = \frac{adj.P}{2} \qquad \because q_{23} = -\frac{k}{8}$

$\therefore -\frac{(3\alpha+4)}{2} = \frac{k}{8} \Rightarrow k = 12\alpha + 16 \qquad ....(i)$

$\therefore |P| = 2k \Rightarrow k = 10 + 6\alpha \qquad ....(ii)$

From (i) and (ii) we get $\alpha = -1, k = 4 \qquad \therefore \alpha^2 + k^2 = 17$

53. α is 7^{th} root of unity

$\Rightarrow 1 + \alpha + \alpha^2 + ... + \alpha^6 = 0,\ p + q = -1$

$pq = \alpha^4 + \alpha^6 + \alpha^5 + \alpha^7 + \alpha^8 + \alpha^7 + \alpha^9 + \alpha^{10} = 3 + (\alpha + \alpha^2 + \alpha^3 + ... + \alpha^6) = 3 + (1) = 2$

$\Rightarrow x^2 + x + 2 = 0$

Both I and II are true and II is the correct explanation

54.

M	A	N	K	I	N	D

$\left(\frac{4\times 6!}{2!}\right) + (5!\times 0) + \left(\frac{4!\times 3}{2!}\right) + (3!\times 2) + (2!\times 1) + (1!\times 1) + (0!\times 0) + 1 = 1492$

$\Rightarrow 1440 + 36 + 12 + 4 = 1492$

55. (A) No. of such triangles = $10\ {}^6C_1 = 60$

(B) No. of such triangles = 10

(C) Number of such quadrilaterals = $10\ {}^5C_1 + = 75$.

(D) Number of such quadrilaterals = 10 (when four consecutive points are taken)

56. Sol: Let $p(E_1) = x, p(E_2) = y$ and $p(E_3) = z$

$\alpha = p(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = p(E_1).p(\bar{E}_2).p(\bar{E}_3)$

$\Rightarrow \alpha = x(1-y)(1-z) \qquad ...(i)$

Similarly,

$\beta = (1-x).y(1-z) \qquad ..(ii)$

$\gamma = (1-x)(1-y).z \qquad ..(iii)$

$p = (1-x)(1-y)(1-z) \qquad ...(iv)$

From (i) and (iv),

$\frac{x}{1-x} = \frac{\alpha}{p} \Rightarrow x = \frac{\alpha}{\alpha + p}$

From (iii) and (iv),

$\frac{z}{1-z} = \frac{\gamma}{p} \qquad \Rightarrow z = \frac{\gamma}{\gamma + p}$

$$\frac{p(E_1)}{p(E_2)}=\frac{x}{z}=\frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}}=\left(\frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}}\right) \quad \ldots(v)$$

Given that

$$(\alpha-2\beta)p=\alpha\beta \Rightarrow \alpha p=(\alpha+2p)\beta \quad \ldots(vi)$$

$$(\beta-3\gamma)p=2\beta\gamma \Rightarrow 3\gamma p=(p-2\gamma)\beta \quad \ldots(vii)$$

From (vi) and (vii),

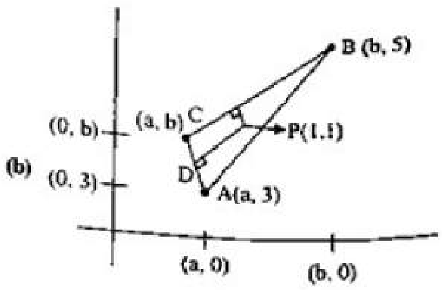
$$\frac{\alpha}{3\gamma}=\frac{\alpha+2p}{p-2\gamma}\Rightarrow p\alpha-6p\gamma=5\gamma\alpha \quad \ldots(viii)$$

$$\Rightarrow \frac{p}{\gamma}-\frac{6p}{\alpha}=5 \quad \Rightarrow \frac{p}{\gamma}+1=6\left(\frac{p}{\alpha}+1\right)$$

From (v) and (viii),

$$\frac{p(E_1)}{p(E_3)}=6$$

57.



Slope of AC = ∞

Slope of PD = 0

$$D\left(\frac{a+a}{2},\frac{b+3}{2}\right)=D\left(a,\frac{b+3}{2}\right)$$

$$\frac{b+3}{2}-1=0; b+3-2=0\Rightarrow b=-1$$

$$\boxed{b=-1}$$

$$E\left(\frac{b+a}{2},\frac{5+b}{2}\right)=\left(\frac{a-1}{2},2\right)$$

Slope of BC × Slope of EP = −1

$$\left(\frac{5-b}{b-a}\right)\times\left(\frac{2-1}{\frac{a-1}{2}-1}\right)=-1$$

$$\Rightarrow\left(\frac{6}{-1-a}\right)\times\left(\frac{2}{a-3}\right)=-1\Rightarrow 12=(1+a)(a-3)$$

$$\Rightarrow 12=a^2-3a+a-3\Rightarrow a^2-2a-15=0$$

$$(a-5)(a+3)=0$$

$$ab>0\Rightarrow a(-1)>0; -a>0; a<0$$

$\boxed{a=-3}$ Accept

Equation of AP $A(-3,3), P(1,1)$

$$y-1=\left(\frac{3-1}{-3-1}\right)(x-1)$$

$-2y+2=x-1$

$\Rightarrow \boxed{x+2y=3}$

Equation of BC $B(-1,5)$, $C(-3,-1)$

$$y+1=\left(\frac{5+1}{-1+3}\right)(x+3)=3x+9$$

$3x-y+8=0$

Solving AP and BC $Q\equiv\left(\frac{-13}{7},\frac{17}{7}\right)$

58. Given equation is

$e^{4x}+4e^{3x}-58e^{2x}+4e^{x}+1=0$

Take, $f(x)=\left(e^{2x}+\frac{1}{e^{2x}}+4\left(e^{x}+\frac{1}{e^{x}}\right)-58\right)$

Let $e^{x}+\frac{1}{e^{x}}=p(>0)$ ......(i)

$p^2+4p-60=0$

$p=6$ or $=-10$

Only $p=6$ is allowed

$$e^{x}+\frac{1}{e^{x}}=6$$

Two real and distinct values of x

59. Since, given $\ell_1+\ell_2=20\Rightarrow\frac{d\ell_2}{d\ell_1}=-1$

Now, $A_1=\left(\frac{\ell_1}{4}\right)^2$ and $A_2=\pi\left(\frac{\ell_2}{2\pi}\right)^2$

Let $S=2A_1+3A_2=\frac{\ell_1^2}{8}+\frac{3\ell_2^2}{4\pi}$

For max or min

$$\frac{ds}{d\ell_1}=0\Rightarrow\frac{2\ell_1}{8}+\frac{6\ell_2}{4\pi}.\frac{d\ell_2}{d\ell_1}=0\Rightarrow\frac{\ell_1}{4}=\frac{6\ell_2}{4\pi}\Rightarrow\frac{\pi\ell_1}{\ell_2}=6$$

60. Let $f(x)=4x^3-11x^2+8x-5\ \forall x\in R$

$\Rightarrow f'(x)=12x^2-22x+8$

$f'(x)=0$

$\Rightarrow 2(6x^2-11x+4)=0\Rightarrow 6x^2-8x-3x+4=0$

$\Rightarrow (2x-1)(3x-4)=0$

$\therefore$ function is decreasing in $\left(\frac{1}{2},\frac{4}{3}\right)$

61. Since, we know $\overline{AB}+\overline{BC}+\overline{CA}=\overline{0}$

$\Rightarrow \alpha=2, \beta=4, \gamma-\delta=3$

Now, $\frac{1}{2}\left|\overline{AB}\times\overline{AC}\right|=5\sqrt{6}$

$(\delta-9)^2+(2\delta+12)^2+100=600 \Rightarrow \delta=5, \gamma=8$

Hence, $\overline{CB}.\overline{CA}=60$

62. Given points and direction ratios are shnown below.

$a_1=(1,2,3), a_2=(2,4,5), \vec{b}_2=2\hat{i}+3\hat{j}+\lambda\hat{k}$

$\vec{b}_2=\hat{i}+4\hat{j}+5\hat{k}$

Apply shortest distance formula,

Shortest distance $=\frac{\left|(a_2-a_1).(b_1\times b_2)\right|}{\left|b_1\times b_2\right|}$

$$\text{S.D.}=\frac{\left|\left((2-1)\hat{i}+(4-2)\hat{j}+(5-3)\hat{k}\right).\left(\vec{b}_1\times\vec{b}_2\right)\right|}{\left|b_1\times b_2\right|} \quad \text{.....(i)}$$

Take, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & \lambda\\ 1 & 4 & 5\end{vmatrix}$

$=\hat{i}(15-4\lambda)+\hat{j}(\lambda-10)+\hat{k}(5)=(15-4\lambda)\hat{i}+(\lambda-10)\hat{j}+5\hat{k}$

$\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}$

From equation (i),

$$\text{S.D.}=\frac{\left|\left(\hat{i}+2\hat{j}+2\hat{k}\right).\left[(15-4\lambda)\hat{i}+(\lambda-10)\hat{j}+5k\right]\right|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}$$

$$\frac{\left|15-4\lambda+2\lambda-20+10\right|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}=\frac{1}{\sqrt{3}}$$

Take square both sides,

$3(5-2\lambda)^2=225+16\lambda^2-120\lambda+\lambda^2+100-20\lambda+25$

$12\lambda^2+75-60\lambda=17\lambda^2-140\lambda+350$

$5\lambda^2-80\lambda+275=0 \Rightarrow \lambda^2-16\lambda+55=0$

$(\lambda-5)(\lambda-11)=0 \Rightarrow \lambda=5,11$

Sum of values of $\lambda=5+11=16$

63. $15\sin^4\alpha+10(1-\sin^2\alpha)^2=6 \Rightarrow 25\sin^4\alpha-20\sin^2\alpha+4=0$

$\Rightarrow 25\sin^4\alpha-10\sin^2\alpha-10\sin^2\alpha+4=0$

$\Rightarrow (5\sin^2\alpha-2)^2=0 \Rightarrow \sin^2\alpha=\frac{2}{5}$

$\therefore \cos^2\alpha=\sqrt{1-\frac{4}{25}}=\frac{3}{5}$

Now, $27\sec^6\alpha+8\operatorname{cosec}^6\alpha=27\left(\frac{5}{3}\right)^3+8\left(\frac{5}{2}\right)^3=125+125=250.$

64. For, S_1 we have

$$\Rightarrow \frac{(x+2)(x^2+3x+5)}{x^2-3x+2} \le 0$$

$$\Rightarrow x \in (-\infty,-2] \cup (1,2)$$

For S_2, we have

$$= 3^x(3^x-3) - 3^2(3^x-3) \le 0$$

For $S_2, x \in [1,2]$

$$\Rightarrow (-\infty,-2] \cup [1,2]$$

65. $$\int \frac{x^2}{(x\sin x+\cos x)^2}dx$$

$$\because \frac{d}{dx}(x\sin x+\cos x) = x\cos x$$

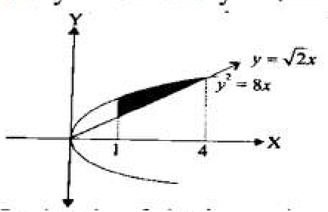
$$\int \underset{II}{\frac{x\cos x}{(x\sin x+\cos x)^2}}\underset{I}{\left(\frac{x}{\cos x}\right)}dx = \frac{x}{\cos x}\left[\frac{-1}{x\sin x+\cos x}\right]$$

$$-\int \frac{x\sin x+\cos x}{\cos^2 x}\left[\frac{-1}{x\sin x+\cos x}\right]dx$$

$$= \frac{x}{\cos x}\left[\frac{-1}{x\sin x+\cos x}\right] + \int \sec^2 x\,dx$$

$$= \frac{-x\sec x}{x\sin x+\cos x} + \tan x + C$$

66. Given equations are $y^2 = 8x$ and $y = \sqrt{2}x$



Put the value of y in other equation

$$\Rightarrow 8x = 2x^2, 2x^2 - 8x = 0$$

$$2x(x-4) = 0 \Rightarrow x = 0 \,\&\, 4$$

Area : $$\int_1^4 \left(2\sqrt{2}\sqrt{x} - \sqrt{2}x\right)dx = 2\sqrt{2}\left(\frac{x^{\frac{3}{2}}}{3/2}\right)_1^4 - \sqrt{2}\left(\frac{x^2}{2}\right)_1^4$$

Apply the limit,

$$= \frac{4\sqrt{2}}{3}(8-1) - \frac{\sqrt{2}}{2}(16-1) = \frac{28\sqrt{2}}{3} - \frac{15\sqrt{2}}{2} = \frac{11\sqrt{2}}{6}$$

67. Given, $(1+x^2)dy = y(x-y)dx$

where, $y(0) = 1, y(2\sqrt{2}) = \beta$

$$dy = \left(\frac{yx-y^2}{1+x^2}\right)dx$$

$$\frac{dy}{dx}+y\left(\frac{-x}{1+x^2}\right)=\left(\frac{-1}{1+x^2}\right)y^2$$

Divide y^2 both side

$$\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{y}\left(\frac{-x}{1+x^2}\right)=\frac{-1}{1+x^2}$$

Put $\frac{1}{y}=t$ then $\frac{-1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$$\Rightarrow \frac{dt}{dx}+t\frac{x}{1+x^2}=\frac{1}{1+x^2}$$

$$\text{I.F.}=e^{\int\frac{x}{1+x^2}dx}=e^{\ln\sqrt{1+x^2}}=\sqrt{1+x^2}$$

$$\Rightarrow t\sqrt{1+x^2}=\int\left(\frac{1}{1+x^2}.\sqrt{1+x^2}\right)dx$$

$$\frac{\sqrt{1+x^2}}{y}=\ln\left(x+\sqrt{x^2+1}\right)+c$$

At, $y(0)=1 \quad \Rightarrow c=1$

$$\Rightarrow \sqrt{1+x^2}=y\ \ln\left(e\left(x+\sqrt{x^2+1}\right)\right)$$

$$\beta=\frac{3}{\text{IN}\left(e\left(3+2\sqrt{2}\right)\right)}\Rightarrow\frac{3}{\beta}=\ln\left(e\left(3+2\sqrt{2}\right)\right)$$

$$e^{\frac{3}{\beta}}=e\left(3+2\sqrt{2}\right)$$

68. First common term of both the series is 23 and common difference is $7\times4=28$

$\because$ Last term ≤ 407

$\Rightarrow 23+(n-1)\times28\le407\Rightarrow(n-1)\times28\le384$

$\Rightarrow n\le\frac{384}{28}+1\Rightarrow n\le14.71$

Hence, number of terms common are 14

69.

C.I.	f_i	x_i	f_ix_i	C.F.
0-6	a	3	3a	a
6-12	b	9	9b	a+b
12-18	12	15	180	a+b+12
18-24	9	21	189	a+b+21
24-30	5	27	135	a+b+26
	N=(26+a+b)		$(504+3a+9b)$	

$$\text{Mean}=\frac{504+3a+9b}{26+a+b}=\frac{309}{22}$$

$\Rightarrow 243a+111b=3054$

$\Rightarrow 81a+37b=1018$ .....(i)

Median class is 12 – 18

Now, Median = $=12+\dfrac{\dfrac{a+b+26}{2}-(a+b)}{12}\times 6=14$

$\Rightarrow \dfrac{a+b+26-2a-2b}{2}=4\Rightarrow a+b=18$ ...(ii)

on solving eqs. (i) and (ii), we get

$a=8, b=10$

$\therefore (a-b)^2=(8-10)^2=4$

70. $\bigcup_{i=1}^{50} X_i=\bigcup_{i=1}^{n} Y_i=T;\ \because n(X_i)=10, n(Y_i)=5$

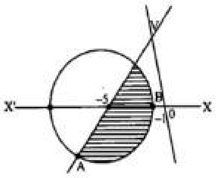
So, $\bigcup_{i=1}^{50} X_i=500, \bigcup_{i=1}^{n} Y_i=5n\Rightarrow \dfrac{500}{20}=\dfrac{5n}{6}\Rightarrow n=30$

71. $z(1+i)+\bar{z}(1-i)\ge -10$

$\Rightarrow (z+\bar{z})+i(z-\bar{z})\ge -10\Rightarrow x-y+5\ge 0$

And $|z+5|\le 4$ is interior of a circle with centre $(-5,0)$ and radius 4.

$\therefore$ $|z+1|$ represents the distance of z from -1.



$|z+1|$ is maximum at A.

On solving equation of circle and line we get

$A\left(-2\sqrt{2}-5,-2\sqrt{2}\right)$

$|z+1|^2=AB^2=\left(2\sqrt{2}+4\right)^2+\left(2\sqrt{2}\right)^2$

$\alpha+\beta\sqrt{2}=32+16\sqrt{2}$

So, $\alpha+\beta=32+16=48$.

72. Given matrix is $A=\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \lambda+\alpha & \alpha+\beta\end{vmatrix}$

Applying, $R_3\rightarrow R_3+R_1$

$\Rightarrow |A|=|\alpha+\beta+\gamma|\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ 1 & 1 & 1\end{vmatrix}$

$\Rightarrow |A|=(\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

$\because |adjA|=|A|^{n-1}$

$|adj(adjA)|=|A|^{(n-1)2}$

$|adj(adj(adj(adjA)))|=|A|^{(n-1)^4}=|A|^{2^4}=|A|^{16}$

$\therefore (\alpha+\beta+\gamma)^{16}=2^{32}.3^{16}$

$\Rightarrow (\alpha+\beta+\gamma)^{16} = (2^2.3)^{16} = (12)^{16}$

$\Rightarrow \alpha+\beta+\gamma = 12$. Hence, $\alpha, \beta, \gamma \in N$

$\Rightarrow (\alpha-1)+(\beta-1)+(\gamma-1) = 9$

Number all tuples $(\alpha, \beta, \gamma) = {}^{11}C_2 = 55$

1 case for $\alpha = \beta = \gamma$

And 12 case when any two of these are equal

So, No. of distinct tuples $(\alpha, \beta, \gamma) = 55-1-12 = 42$

73. Since $54 = 3^3 \times 2$

Given that number whose G.C.D with 54 is 2.

$\therefore$ Numbers should be divisible by 2 but not by 3

N = (Numbers divisible by 2) – (Number divisible by 6)

$$N = \frac{9000}{2} - \frac{9000}{6} = 4500 - 1500 = 3000$$

74.

$$0 \le \frac{1-d}{4} \le 1 \quad \Rightarrow \quad -3 \le d \le 1 \qquad ..(i)$$

$$0 \le \frac{1+2d}{4} \le 1 \quad \Rightarrow \quad -\frac{1}{2} \le d \le \frac{3}{2} \qquad ...(ii)$$

$$0 \le \frac{1-4d}{4} \le 1 \quad \Rightarrow \quad -\frac{3}{4} \le d \le \frac{1}{4} \qquad ...(iii)$$

$$0 \le \frac{1+3d}{4} \le 1 \quad \Rightarrow \quad -\frac{1}{3} \le d \le 1 \qquad ...(iv)$$

From (i), (ii), (iii) and (iv)

$-\frac{1}{3} \le d \le \frac{1}{4}$ Minimum value of $d = -\frac{1}{3}$

$$\text{Mean} = 0 + \frac{1+2d}{4} + \frac{2(1-4d)}{4} + \frac{3(1+3d)}{4}$$

$$\overline{X} = \frac{6+3d}{4} = \frac{1}{4}\left(6 - 3\times\frac{1}{3}\right) = \frac{5}{4} \Rightarrow 60\overline{X} = 60 \times \frac{5}{4} = 75$$

75. Given system of equations are $x+y+z=6$

$2x+5y+\alpha z = \beta$

$x+2y+3z=14$

From the given equations.

$x+y = 6-z$ ....(i)

$x+2y = 14-3z$ ....(ii)

Subtract (i) from (ii),

$\Rightarrow y = 8-2z$, then x = z-2.

Now, put the values x & y

In eq. $zx+5y+\alpha z = \beta$.

$2(z-2)+5(8-2z)+\alpha z = \beta$

$(\alpha-8)z = \beta-36$

For having infinite solutions

$\alpha-8=0 \& \beta-36=0$

$\alpha=8, \beta=36$

Required sum = $\alpha+\beta = 44$